Analog Design
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Widlar Current Source
Closed Form Solution

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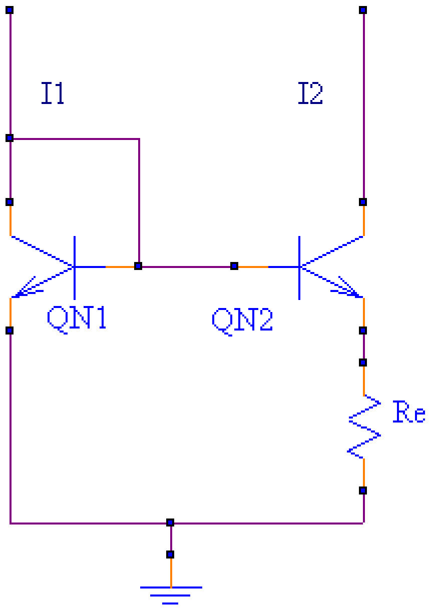
Abstract

This paper derives a closed form solution for the Widlar current mirror. Traditionally, it is often stated that the equation is transcendental with no solution in terms of other known mathematically functions. This paper shows that the solution can be expressed in closed form using relatively well known mathematical functions.

Analysis

The Widlar current source is shown in fig 1.



Nodal loops can be set up to find I1 as a function of I2.

First, using the well known exponential relation for transistor current as a function of base emitter voltage.

$$i_c = i_o e^{\frac{V_{be}}{Vt}}$$

or

$$V_{be} = V_t \log(\frac{i_c}{i_o})$$

Using these relations, the following can be derived from inspection

$$V_{beq1} = V_t \log(\frac{i_1}{i_o}) = V_t \log(\frac{i_2}{i_o}) + I_2 R_e$$

$$V_t((\log(\frac{i_1}{i_o}) - \log(\frac{i_2}{i_o})) = I_2 R_e$$

$$V_t \log(\frac{i_1}{i_o}\frac{i_o}{i_2}) = I_2 R_e$$

$$V_t \log(\frac{i_1}{i_2}) = I_2 R_e$$

$$\log(\frac{i_1}{i_2}) = \frac{I_2 R_e}{V_t}$$

$$\frac{i_1}{i_2} = e^{\frac{I_2 R_e}{V_t}}$$

$$i_1 = i_2 e^{\frac{I_2 R_e}{V_t}} \quad \text{- 1}$$

This equation allows I1 to be calculated when I2 is given. However, if it is desired to determine I2 when I1 is known, it is a bit trickier. Fortunately this particular problem has already been solved.

Consider the following equation.

$$x = ye^y \quad \text{-2}$$

This equation turns up quite a lot, in particular in evolution analysis, such that it has been extensively studied and is a standard function available in most mathematics software. The solution for y is written as:

$$y = W(x) \quad \text{-3}$$

where W(x) is the *Lambert W Function.*

In (1) let:

$$y = \frac{I_2 R_e}{V_t} \quad -4$$

Then (2) becomes

$$i_1 = \frac{yV_t}{R_e} e^y \quad - 5$$

or

$$\frac{i_1 R_e}{V_t} = ye^y \quad - 6$$

Therefore in terms of W(x), the solution for y is:

$$y = W(\frac{i_1 R_e}{V_t})$$

or by back substituting for y

$$\frac{I_2 R_e}{V_t} = W(\frac{i_1 R_e}{V_t})$$

therefore:

$$I_2 = \frac{V_t}{R_e} W(\frac{i_1 R_e}{V_t}) \quad -7$$

Is the closed form solution for I2 in terms of the Lambert W Function.

V supply Resistor Diode Circuit

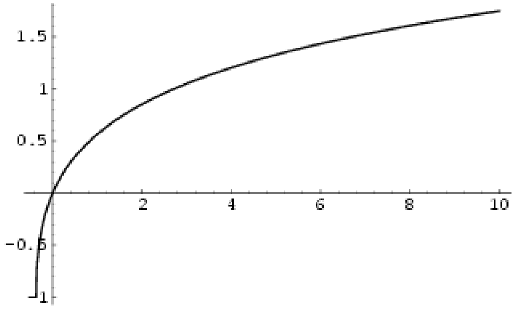
With a little more work, and as an exercise for the reader, a series circuit consisting of a supply, resistor and diode will satisfy:

$$Id = \frac{V_t}{R} W(\frac{i_s R}{V_t} e^{\frac{Vs}{V_t}}) \quad -8$$

Appendix A

Lambert W Function

For reference, the following are noted.



courtesy of Wolfram Research - http://mathworld.wolfram.com/LambertsW-Function.html, from which "Mathematica" can of course be obtained.

$$W(x) = \sum_{n=1}^{n=\infty} \frac{(-n)^{n-1}}{n!} x^n$$

There are better convergent series then this one, so the interested reader should look up other suitable references if required.

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