# General Relativity For Teletubbies

**Sir Kevin Aylward B.Sc., Warden of the King's Ale**

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## Galilean Transformation

## Of

## Maxwell's Equations

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**Abstract**

*There are often claims that Maxell's Equations predict that the speed of light is invariant irrespective of source or observer motion. The argument pretty much, invariably, states that the EM wave equation has a velocity given by:*

$$c = \frac{1}{\sqrt{\varepsilon\mu}}$$

*With the claim that, because there are no other velocity terms in the expression, the velocity is not with respect to anything. These claims are usually the result of the claimers simply copying from those that copied, that copied, down a long chain, with no evaluation as to whether such a claim is even rational. The claim is false, and it is certainly not controversial that such a claim is false. Indeed the considerable effort in engaging in the Michaelson-Morley Experiment is testament to this. The MMX was specifically designed to detect a change in light's velocity in relation to the observers velocity via way of the Earth's motion through the Solar system, principally because that is what Maxwell's Equations indicated would be the case.*

*However, if as an independent assumption, Maxwell's Equations are subject to the Lorentz Transformations, then they do "predict" an invariant velocity of light.*

*The following is a non-original summary derivation of the expected change in lights' velocity according to the Galilean Transformation applied to Maxwell's Equations.*

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**Galilean Transformation of Maxwell's Equation**

In order to actually determine what Maxwell's Equations predict about observers moving relative to EM fields, one has to *actually calculate* what the wave equation will be from Maxwell's equations under a Galilean Transformation.

The core of this analysis was taken from:

**https://physics.stackexchange.com/questions/378861/what-does-a-galilean-transformation-of-maxwells-equations-look-like**

For two reference frames **S, S**', with one moving with relative with respect to another, the Galilean Transformation is given by:

$$t' = t$$

$$\mathbf{x}' = \mathbf{x} - \mathbf{v}t$$

The full Lorentz transformation of the Electromagnetic Fields and Sources, as provided by the Wikipedia article:

https://en.wikipedia.org/wiki/Classical_electromagnetism_and_special_relativity

are:

$$\mathbf{E}' = \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - (\gamma - 1)(\mathbf{E} \cdot \hat{\mathbf{v}})\hat{\mathbf{v}}$$

$$\mathbf{B}' = \gamma(\mathbf{B} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}) - (\gamma - 1)(\mathbf{B} \cdot \hat{\mathbf{v}})\hat{\mathbf{v}}$$

$$\mathbf{J}' = \mathbf{J} - \gamma\rho\mathbf{v} + (\gamma - 1)(J \cdot \hat{\mathbf{v}})\hat{\mathbf{v}}$$

$$\rho' = \gamma(\rho - \frac{J.\mathbf{v}}{c^2})$$

The Galilean Transform of the fields and sources are thus obtained by taking the limit of $c \to \infty$ so that $\gamma \to 1$, which results in:

$$\mathbf{E} = \mathbf{E}' - \mathbf{v} \times \mathbf{B}'$$

$$\mathbf{B} = \mathbf{B}'$$

$$\mathbf{J} = \mathbf{J}' + \rho'\mathbf{v}$$

$$\rho = \rho'$$

After swapping $\mathbf{v} \to -\mathbf{v}$

In the frame **S**, Maxwell's Equations are:

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t})$$

Thus, in the transformed frame **S'**, Maxwell's Equations are:

$$\nabla \cdot (\mathbf{E}' - \mathbf{v} \times \mathbf{B}') = \rho' / \epsilon_0$$

$$\nabla \cdot \mathbf{B}' = 0$$

$$\nabla \times (\mathbf{E}' - \mathbf{v} \times \mathbf{B}') = -\frac{\partial \mathbf{B}'}{\partial t}$$

$$\nabla \times \mathbf{B}' = \mu_0 (\mathbf{J}' + \rho' \mathbf{v} + \epsilon_0 \frac{\partial (\mathbf{E}' - \mathbf{v} \times \mathbf{B}')}{\partial t})$$

With the derivatives also transformed by:

$$\nabla = \nabla'$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \mathbf{v} \cdot \nabla$$

And the primes throughout, simply for convenience, now swapped to non primes, results in the transformed Maxwell's Equations of:

$$\nabla \cdot \mathbf{E} + \mathbf{v} \cdot (\nabla \times \mathbf{B}) = \rho / \epsilon_0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{J} + \rho \mathbf{v} + \epsilon_0 \frac{\partial}{\partial t} (\mathbf{E} - \mathbf{v} \times \mathbf{B}) - \epsilon_0 \mathbf{v} \cdot \nabla (\mathbf{E} - \mathbf{v} \times \mathbf{B}))$$

Taking the curl of the last equation, and substituting for **E** from the 1st and 3rd equations gives the modified wave equation for **B**:

$$c^2\nabla^2\mathbf{B} = \frac{\partial^2\mathbf{B}}{\partial t^2} + (\mathbf{v}\cdot\nabla)^2\mathbf{B} - 2\mathbf{v}\cdot\nabla(\frac{\partial\mathbf{B}}{\partial t})$$

Whence a solution of the form

$$\mathbf{B} = B_o \exp i(\mathbf{k}\cdot\mathbf{x} - \omega t)$$

is obtainable with

$$\omega = -\mathbf{v}\cdot\mathbf{k} \pm c\,|\,\mathbf{k}\,|$$

resulting in a group velocity for the wave of:

$$\frac{\partial\omega}{\partial\mathbf{k}} = -\mathbf{v} \pm c\hat{\mathbf{k}}$$

which results in a change in velocity :

$$c \pm \mathrm{v}$$

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http://www.kevinaylward.co.uk/gr/index.html

www.kevinaylward.co.uk

kevinEXTRACTextract@kevinaylward.co.uk

Remove EXTRACT from the email address

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