Gravitational Mass-Energy

Without GR?

A Food for Thought

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Overview

The following is an attempt to extend on the idea of the derivation of $E=mc^2$ without using the Lorentz Transform, thus it is recommended that that approach be first reviewed.

The approach used here produces an equivalent result of mass variation for the instance of gravitational potential energy rather than kinetic energy. The principle illustrated is the often quoted result from general relativity, that winding up a mechanical watch increases its mass by way of increasing its potential energy.

Its intent is to give a clearer insight into the approach of General Relativity where mass-energy is indeed rather flexible and difficult to actually pin down. The simplified argument used here, although interesting, does result in a contradiction to General Relativity and experimental evidence. However, the road to truth always takes many false paths on the way…

Derivation

The initial assumption is made that there is a small test mass $\mathbf{m_r}$, that is subject to a gravitational field from a much larger mass $\mathbf{M_g}$ such that any change in mass of the test mass $\mathbf{m_r}$ as it is moved with respect to the larger mass generating the gravitational field, is reflected in the test mass and not the mass $\mathbf{M_g}$ generating the potential. This assumption is very problematic, but allows an initial simplified argument to be constructed. It is expected that reformulating with both masses subject to variation would lead to an improved result

This assumption alone illustrates the general problem in General Relativity of identifying how to define mass. Mass is not a unique property of an object, but a relationship between all other masses in the universe. Mass can, in principle, be assigned in a somewhat arbitrary way because energy and mass are equivalent.

The starting points are the Newtonian gravitational equations:

$$F = \frac{Gm_r M_g}{r^2} \quad \text{- (1)}$$

$$PE = \int F dr \quad \text{- (2)}$$

If one assumes that the potential energy of a mass, when it is moved in a gravitational potential, is contained by a change in its mass, one can write, as for the kinetic energy case, in very general terms:

$$PE = k^2 m(r) + \alpha \quad \text{- (3)}$$

Where m(v), the mass, is an arbitrary function of position r, k^2 and α are arbitrary constants.

One can write, simply to the KE example:

$$PE = k^2 m(r) + \alpha = \int \frac{G m_r M_g}{r^2} dr \text{ - (4)}$$

Differentiating both sides of the equation w.r.t:

$$k^2 m'(r) = \frac{G m_r M_g}{r^2} \text{ - (5)}$$

$$\frac{m_r'}{m_r} = \frac{G M_g}{k^2 r^2} \text{ - (6)}$$

This equation can be immediately integrated:

$$\ln(\beta m_r) = -\frac{G M_g}{k^2 r} \text{ - (7)}$$

$$m_r = m_{r0} e^{-\frac{G M_g}{k^2 r}} \text{ - (8)}$$

Noting that at r = infinity, $\mathbf{m_r = m_{r0}}$

Although it is not possible to show that k is c, the speed of light in this derivation, for the purposes of this paper it will be taken as such, so that:

$$m = m_{r0} e^{-\frac{G M_g}{c^2 r}} \text{ - (9)}$$

This equation holds that an object’s effective mass at infinity, is reduced as it gets further from a gravitational source. Substituting into the force equation results in:

$$F = \frac{G M_g m_{r0} e^{-\frac{G M_g}{c^2 r}}}{r^2} \text{ - (10)}$$

Which has the somewhat interesting property that in the limit as r->0, the force is zero, due to the rate that the exponential term goes to zero faster than the $1/r^2$ term does. Somewhat at odds to the results of General Relativity’s black hole dynamics though ☺….

The PE is given by integrating the force:

$$PE = -c^2 m_{r0}(1 - e^{-\frac{GM_g}{c^2 r}})$$ - (11)

Substituting back into to - (3), with suitable initial conditions will give:

$$PE = c^2(m - m_o)$$ - (12)

or

$$PE = c^2 \Delta m$$ - (13)

Which is in accord with the results from General Relativity

Expanding the mass equation exponential to first order results in:

$$m = m_{r0}(1 - \frac{GM_g}{c^2 r})$$ - (14)

Expanding the potential energy exponential to second order results in:

$$PE = -c^2 m_{r0}(1 - (1 - \frac{GM_g}{c^2 r} + \frac{1}{2}(\frac{GM_g}{c^2 r})^2))$$ - (15)

$$PE = -\frac{GM_g m_{r0}}{r}(1 - \frac{GM_g}{2c^2 r})$$ - (16)

This is the somewhat problematic issue alluded to at the introduction. The correction to the Newtonian potential energy is only ½ of that required to account for gravitational redshift.

Looks like the original Einstein calculation that resulted in ½ of the correct value has popped up once again….

I will leave it as an exercise for the reader to determine if allowing both masses to vary, fixes the problem….

Website last modified 3rd December 2023

http://www.kevinaylward.co.uk/gr/index.html

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