# General Relativity For Tellytubbys

# The Special Relativity Section



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**Overview**

Special Relativity (S.R.) has probably spawned more responses from individuals then a naked Heather Locklear strolling through central park (sci.physics.relativity). The main reason for this section is that it is required for an understanding of the stress-energy tensor.

You can get a bit more of an understanding for the rational of SR here SR Background.

So, straight to the point. It is noted that, independent of whether light is treated as a wave, particle or Teletubby, and independent of ones velocity with respect to the light source, the speed of light is invariant in vacume. That is, it is always measured, or all experiments can only be understood, if the speed of light always remains constant. This is of course very queer. No matter how fast one travels into the sun, its light will always be measured to have the same speed.

Secondly, various results of physics don't seem to depend on what does the moving, only relative velocities seem to matter.

These are expressed as:

1) *The Principle Of Relativity (POR) Postulate/Axiom*

*The Laws Of Physics are independent of inertial motion (non accelerated motion).*

2) *The Speed of Light (SOL) Postulate/Axiom:*

*The SOL, in a vacuum, is an invariant. That is, the SOL is always measured to be the same irrespective of the source of the light's velocity or the observers velocity.*

Postulate 1 is quite often expressed or explained quite incorrectly. The essential bit though is that it is just as valid to swap over what object is stationary and what object is moving. That is, there is no concept of absolute rest, all uniform motion is equivalent. It is important to point out there is nothing new in this postulate from Newtonian Mechanics.

The 1st Postulate fundamentally means that *the forces between co-moving objects are independent of their joint velocity with respect to other inertial frames*.

**The Sums of the Lorentz Transformation**

Everyone got to the equations before Einstein, but somehow he managed to steal their thunder. The Lorentz Transforms were first published in Playboy, but by someone who, for obvious reasons, wanted to remain anonymous. So because of this Lorentz became the fall guy, and in fact he only became aquatinted with them whilst waiting for the number 133 bus at Dagenham East, while "reading" the news section, or so he claimed.

Here we go. Lets pretend to derive the co-ordinate transform equations of SR. Imagine standing in a box and viewing another a box moving by you, left to right (the x direction) at velocity v, that's our co-ordinate systems described then, i.e. where the origins are the front left corners of the boxes. The moving systems origin is called O', the stationary (reference) one O. If the reference frame emits a pulse of light from its origin, the pulse's position can be expressed by:

$$x^2 + y^2 + z^2 = c^2 t^2$$

Since the pulse's radial position is ct

or

$$x^2 + y^2 + z^2 - c^2 t^2 = 0$$

For the moving box, its equation for the light pulse would be:

$$\bar{x}^2 + \bar{y}^2 + \bar{z}^2 = c^2 \bar{t}^2$$

or

$$\bar{x}^2 + \bar{y}^2 + \bar{z}^2 - c^2 \bar{t}^2 = 0$$

Where, by assumption the t bar might be different, and it is to be determined if that is indeed the case. Note, our speed of light postulate requires that c is the same in both sets of equations.

These equations can then be equated thus:

$$x^2 + y^2 + z^2 - c^2 t^2 = \bar{x}^2 + \bar{y}^2 + \bar{z}^2 - c^2 \bar{t}^2$$

And, to cut a long story short, a bit of rationalization will indicate that the y's and z's will in both systems will be the same, thus:

$$x^2 - c^2 t^2 = \bar{x}^2 - c^2 \bar{t}^2$$

The bit to notice here is that both sides of the equation are an invariant. i.e. its value don't change with the inertial frames motion.

So, how are the x an x bars related. One assumes for all sorts of high-brow reasons, that the coordinates are related by a linear transform:

$$\bar{x} = \gamma(x - avt)$$

$$\bar{t} = \gamma'(t - \frac{bv}{c^2} x)$$

These might look a bit contrived, but this simply states that the x bar and t bar are linear functions of x and t, the constants are arbitrary and to be determined, and all come out in the wash.

Lets see what the relativity postulate gets us first.

Well, it means that we can simply swap over the x to x bar and t tot bar and change the sign of the velocity, thus

$$x = \gamma(\bar{x} + av\bar{t})$$

$$t = \gamma'(\bar{t} + \frac{bv}{c^2}\bar{x})$$

Now if we substitute the above pair of equations into the above, above x equation…

$$\bar{x} = \gamma(\gamma(\bar{x} + av\bar{t}) - av\gamma'(\bar{t} + \frac{bv\bar{x}}{c^2}))$$

equating the coefficients of t bar:

$$0 = \gamma' av - av\gamma'$$

hence

$$\gamma' = \gamma$$

Using this result and equating the coefficients of x bar:

$$1 = (\gamma - \frac{abv^2}{c^2}\gamma)\gamma$$

hence

$$\gamma = \frac{1}{\sqrt{1 - \frac{abv^2}{c^2}}}$$

now from above

$$x^2 - c^2t^2 = \bar{x}^2 - c^2\bar{t}^2$$

Substituting into the above gets us:

$$x^2 - c^2t^2 = \gamma^2(x - avt)^2 - c^2\gamma^2(t - \frac{bvx}{c^2})^2$$

$$x^2 - c^2t^2 = \gamma^2(x^2 - 2avxt + a^2v^2t^2) - c^2\gamma^2(t^2 - \frac{2bvxt}{c^2} + \frac{b^2v^2x^2}{c^4})$$

For this to be true for all x and t the coefficients of each power of x and t must be equal:

xt term,

$$0 = 2av - 2bv$$

or

$$a = b$$

$x^2$ term

$$1 = \gamma^2 (1 - \frac{b^2 v^2}{c^2})$$

$t^2$ term

$$1 = \gamma^2 (1 - \frac{a^2 v^2}{c^2})$$

Putting it all together gives us.

$$\bar{x} = \gamma(x - vt)$$

$$\bar{t} = \gamma(t - \frac{vx}{c^2})$$

$$where \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

And, one must stress the importance of the second term in the time equation. It's the relativity of simultaneity term that makes SR actually work. It says time is also a function of distance separation.

**<u>Time dilation</u>**

How does time change in the moving system relative to the stationary, one?

$$\frac{d\bar{t}}{dt} = \frac{d}{dt}(\gamma(t - \frac{vx}{c^2}))$$

$$\frac{d\bar{t}}{dt} = \gamma$$

**<u>Length Contraction</u>**

$$\frac{dx}{d\bar{x}} = \frac{1}{\gamma}$$

**<u>Standard Results</u>**

The assumption here is that the reader will get the main SR stuff from elsewhere, this is really a refresher bit just to form a reference for the GR bits and pieces.

Instead of the conventional 3D space and 1D-time descriptions, in SR objects are reformulated in one 4-D description. For example, 3 D velocities are replaced by a 4-d velocity vector called the 4 velocity, u.

**<u>4-position</u>**

$$\mathbf{X} = [ct, x, y, z] = x^\alpha \mathbf{e}_\alpha$$

**<u>4 - velocity</u>**

$$\frac{d\mathbf{x}}{d\tau} = \frac{dx^\alpha}{d\tau}, \alpha \neq 0,$$

where the 0'th term of **u** is time t. Tau is the proper time. Such that:

$$u^0 \equiv c\frac{dx^0}{d\tau} = c\frac{dt}{d\tau} = \gamma c$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} = \gamma\frac{dx}{dt} = \gamma v^\alpha$$

Where alpha = 1,2,3

or

$$\mathbf{u} = [c\gamma, \gamma\dot{x}, \gamma\dot{y}, \gamma\dot{z}]$$

**4 - Momentum**

$$\mathbf{p} = m\mathbf{u}$$

where it is noted that the 0th momentum component is:

$p^0 = mc\gamma = \frac{E}{c}$ as $E = m\gamma c^2$ where $E \equiv KE + mc^2$

Or

$$\mathbf{p} = [\frac{E}{c}, \gamma m v^x, \gamma m v^y, \gamma m v^z]$$

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