Twins Paradox

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Overview

Yet another *true* resolution of the Twins Paradox…

The twins paradox is the notion that if the Principle of Relativity (POR) is valid, then if one twin jaunts off in a rocket to the star Alpha Centauri at a speed close to the velocity of light and returns, it is concluded that there is ambiguity in what twin shows the least age. The argument being that Special Relativity (SR) states that the observed clock ticks of a clock in a frame moving relatively to a clock in a notional stationary frame, are larger, such that time (number of clock ticks) for the moving clock passes slower than the non-moving clock. However, it is also stated that the traveller can consider himself fixed in space, such that the stay at home twin may be considered to be moving such that the stay at home twin can claim to be the younger twin.

There are many accounts of claims of resolving the twins paradox of Special Relativity such as whether acceleration is required, for example trips through “space-time” and some claiming that it is due to switching the direction of frames for the traveller that the stay at home does not experience.

These explanations are not correct. Fundamentally, they have lost the plot.

Neither frame ***switching*** or acceleration form the root cause as to why the traveller is younger.

The root cause is that the ***stay at home twin and the star*** *are both in a* ***different*** *frame from the travelling twin.*

The frames are different because the ***star always stays in the same frame as the stay at home twin****, whatever frame is taken to be at rest*. The times in different frames are different because ***time in frames is dependent on distance as well as time*** of other frames. This is absolutely fundamental to the resolution of the paradox. This changes the distances that the traveller measures from that which the stay at home twin measures. The fact that frame times depend on distance is typically ignored.

Thus the root cause of the asymmetry in times of the twins is:

1 The stay at home twin measures event times at two different locations.

2 The traveller, considered at rest, measures event times at one location.

Whether or not there is a paradox, is whether or not a *correct* application of the Lorentz Transform results in the same results for the time of the trip, independent of who is considered at rest. Hand waving descriptions typically ignore what the true physics actually says. Typically most alleged resolutions don’t actually show the calculations of both viewpoints, they engage in a Strawman that notionally appears to do this, but doesn’t.

Indeed, whether the moving clock is outward or inward makes no difference. According to **a *correct SR calculation, both twins will agree as to the time difference between the start event and end events of even the single way trip, and that the traveller is the youngest***, eliminating the paradox.

Key to this calculation is:

1 Time in a frame is *not* simply:

$$t^{'} = \gamma t \qquad - 1$$

It is:

$$t^{'} = \gamma (t - \frac{vx}{c^2}) \qquad - 2$$

That is, time events in inertial frames are dependent on *both* time events in the frames *and* distance travelled in that frame.

2 Distances (lengths) in frames are *not* the same, they are related by:

$$x^{'} = \gamma x \qquad - 3$$

<u>The System</u>

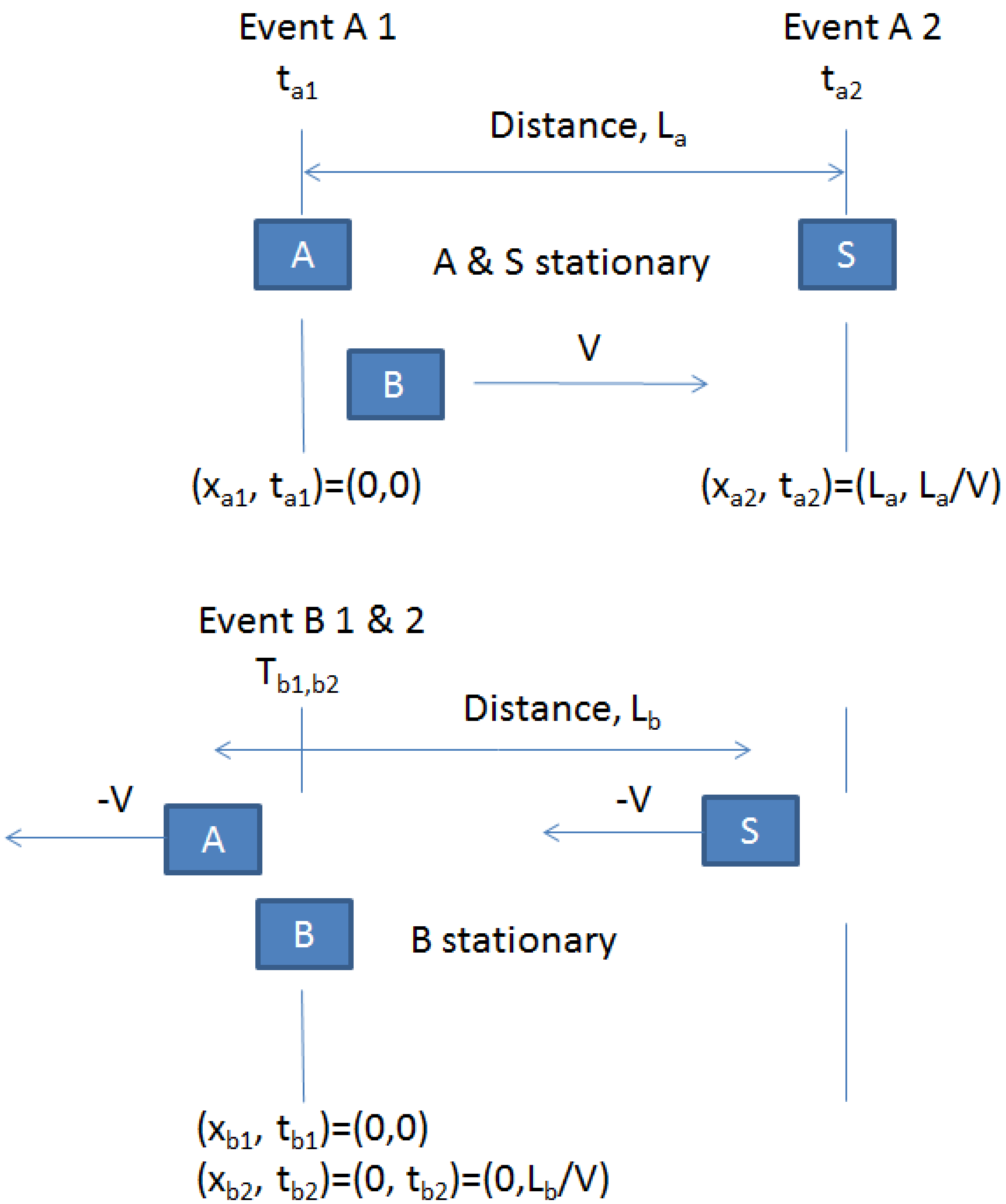

Diagram 1 Traveller Model

A = Stay at home twin

B = Travelling twin

S = Star

L_a = Rest frame distance of stay at home twin to the star, as measured by the stay at home twin

L_b = Rest frame distance of traveller twin measured for distance of stay at home twin to the star

Event 1 = *Time & space coordinates* of when B and A are at the same location

Event 2 = *Time & space coordinates* of when B and S are at the same location

The coordinates should be clear from simple inspection. It takes a time of L/V to get to the star. The time coordinate for the case where B is considered stationary is simply B's own clock time.

The Calculations

The Lorentz Transform (LT) allows the *time and space coordinates of events in one frame* to be calculated from *time and space coordinates of events in another frame*. That is:

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{vx}{c^2})$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{- 4}$$

$$\frac{1}{\gamma^2} = (1 - \frac{v^2}{c^2})$$

Thus, given the coordinates of events 1 & 2 according to A, then the coordinates of B for events 1 & 2 can be calculated as:

$$x_{b1} = \gamma(x_{a1} - vt_{a1})$$

$$t_{b1} = \gamma(t_{a1} - \frac{vx_{a1}}{c^2}) \qquad \text{- 5}$$

$$x_{b2} = \gamma(x_{a2} - vt_{a2})$$

$$t_{b2} = \gamma(t_{a2} - \frac{vx_{a2}}{c^2}) \qquad \text{- 6}$$

The Usually Included Calculation

For diagram 1, the time A calculates for the trip from event 1 to 2, is event 2 time – event 1 time is:

$$t_{a12} = t_{a2} - t_{a1} = (\frac{L_a}{v} - 0) = \frac{L_a}{v} \qquad \text{- 7}$$

Also from diagram 1

$$(x_{a1}, t_{a1}) = (0,0)$$
$$(x_{a2}, t_{a2}) = (L_a, \frac{L_a}{v}) \quad \text{- 8}$$

Thus the coordinates of B for event 1 are:

$$(x_{b1}, t_{b1}) = \gamma(0 - v \times 0, 0 - \frac{v \times 0}{c^2}) = (0,0) \quad \text{- 9}$$

The coordinates of B for event 2 are:

$$(x_{b2}, t_{b2}) = \gamma(L_a - v \times \frac{L_a}{v}, \frac{L_a}{v} - \frac{v \times L_a}{c^2}) = \gamma(0, \frac{L_a}{v}(1 - \frac{v^2}{c^2})) = (0, \frac{L_a}{\gamma v}) \quad \text{- 10}$$

Thus A concludes that the time B calculates for the trip from event 1 to 2, is event 2 time – event 1 time is:

$$t_{b21} = t_{b2} - t_{b1} = (\frac{L_a}{\gamma v} - 0) = \frac{L_a}{\gamma v} = \frac{t_{a21}}{\gamma} \quad \text{- 11}$$

That is:

$$t_{b21} = \frac{t_{a21}}{\gamma} \quad \text{- 12}$$

<u>The Usually Ignored Calculation</u>

Now… the bit that is pretty much always missed missed…*what does B actually calculate for the time A experiences between event 1 and event 2, not what is ad-hoc claimed?*

To do this, one needs to calculate A’s coordinates, given B’s coordinates, that is:

$$x_{a1} = \gamma(x_{b1} + vt_{b1})$$
$$t_{a1} = \gamma(t_{b1} + \frac{vx_{b1}}{c^2}) \quad \text{- 13}$$

Noting the change in the direction of the motion as viewed by B.

The coordinates of B, from the diagram are:

$$(x_{b1}, t_{b1}) = (0,0)$$
$$(x_{b2}, t_{b2}) = (0, t_{b2}) \quad \text{- 14}$$

Thus the coordinates of A for event 1 are:

$$(x_{a1},t_{a1})=\gamma(0+v\times 0,0+\frac{v\times 0}{c^2})=(0,0) \quad - 15$$

The coordinates of A for event 2 are:

$$(x_{a2},t_{a2})=\gamma(0+v\times t_{b2},t_{b2}+0)=\gamma(L_b,t_{b2}) \quad - 16$$

Thus B concludes that A's distance to the star, and its time difference between events is:

$$(x_{a21},t_{a21})=\gamma(L_b,t_{b2})-(0,0)=\gamma(L_b,t_{b2}) \quad - 17$$

That is, B concludes that A's length and time is:

$$\begin{aligned} x_{a21} &= \gamma L_b \\ t_{a21} &= \gamma t_{b21} \end{aligned} \quad - 18$$

That is, B concludes, by simple algebra, that:

$$\begin{aligned} L_b &= \frac{x_{a21}}{\gamma} = \frac{L_a}{\gamma} \\ t_{b21} &= \frac{t_{a21}}{\gamma} \end{aligned} \quad - 19$$

Thus the time that B concludes A measures for B's events, is exactly the same as that A (eq. 12) concludes B measures for B's events, both agree that B reads less time, thus there is no paradox.

That is, both A & B conclude that making a one way trip to the star results in less time for B. Returning just results in doubling up the time, as can be easily calculated simply by resetting t=0 at the star and performing, the same calculation with velocities swapped.

The crucial point is that B views the distance from A to the star as shorter, thus B views events that are synchronised by that length, take less time. Thus despite a notional symmetrical γ in the transform equations when inverting viewpoints, γ is not the sole determinator of the time between events.

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http://www.kevinaylward.co.uk/gr/index.html

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